東吳大學　九十四 學年度碩士班研究生招生考試試題

系級	資訊科學系碩士班A組	考試時間	100 分鐘
科目	離散數學	本科總分	100 分

1 (10%) How many integer solutions for the equation

$$x_1 + x_2 + x_3 + x_4 = 8$$

2 (15%) According to the following statements,

> If the band could not play rock and blue music or the refreshments were not delivered on time, then the party would have been canceled and Mary would have been angry. If the party were canceled, then refunds would have had to be made. No refunds were made.

whether the band could play rock music or not? prove it.

3 (15%) Prove that

For every $n \in Z$, $n \geq 0$, the number $\frac{n^7}{7} + \frac{n^3}{3} + \frac{11n}{21}$ will be an integer.

4 (15%) Suppose $A = \{a_1, a_2, \ldots, a_m\}$ and $B = \{b_1, b_2, \ldots, b_n\}$, prove that the number of one-to-one functions from A to B is $P(n,m) = \frac{n!}{(n-m)!}$.

5 (15%) Given a square ABCD with $\overline{AB} = 1$, please show that if we select five points in the interior of this square, there are at two points whose distance apart is less than $1/\sqrt{2}$.

6 (15%) For $n \in Z^+$, solve the p_n in the following recurrence relation:

$$\begin{aligned} p_1 &= 1 \\ p_2 &= 2 \\ p_n &= 2p_{n-2}, \quad n \geq 3 \end{aligned}$$

7 (15%) What is the length of a possible longest path in a bipartite graph $K_{m,n}$, where $m, n \in Z^+$ with $m < n$. And how many paths of longest length are there in the graph. Note that a path has no direction such that paths $a \to b$ and $b \to a$ are the same paths.