系級	資訊科學系碩士班 A 組	考試時間	100 分鐘
科目	離散數學	本科總分	100 分

1. An urn contains 15 balls, 8 of which are red and 7 are black. In how many ways can 5 balls be chosen so that　(3/each; total: 12)
   (a) all 5 are red?
   (b) all 5 are black?
   (c) 2 are red and 3 are black?
   (d) 3 are red and 2 are black?

2. A die is tossed and the number showing on the top face is recorded. Let E, F, and G be the following events.
   　　　E: The number is at least 3.
   　　　F: The number is at most 3.
   　　　G: The number is divisible by 2.　(3/each; total: 12)
   (a) Are E and F mutually exclusive? Justify your answer.
   (b) Are F and G mutually exclusive? Justify your answer.
   (c) Is $E \cup F$ the certain event? Justify your answer.
   (d) Is $E \cap F$ the impossible event? Justify your answer.

3. In (a) through (e), determine whether the relation R on the set A is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive.　(3/each; total: 15)
   (a) $A = Z$; $a\,R\,b$ if and only if $a \le b+1$.
   (b) $A = Z^+$; $a\,R\,b$ if and only if $a = b^k$ for some $k \in Z^+$.
   (c) $A = Z$; $a\,R\,b$ if and only if $|a - b| = 2$.
   (d) $A = Z^+$; $a\,R\,b$ if and only if $GCD(a, b)=1$, that is, a and b are relatively prime.
   (e) $S = \{1, 2, 3, 4\}$, $A = S \times S$; $(a, b)\,R\,(c, d)$ if and only if $ad = bc$.

4. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$ be a permutation of A.　(3/each; total: 12)
   (a) Write P as a product of disjoint cycles.
   (b) Compute P^{-1}.
   (c) Compute P^2.
   (d) Find the period of P, that is, the smallest positive integer k such that $P^k = 1_A$.

5. Let $A = \{x \mid x$ is a real number and $0 < x < 1\}$, $B = \{x \mid x$ is a real number and $x^2 + 1 = 0\}$, $C = \{x \mid x = 4m, m \in Z\}$, $D = \{(x, 3) \mid x$ is an English word whose length is 3$\}$, and $E = \{x \mid x \in Z$ and $x^2 \le 100\}$. Identify each set as finite, countable, or uncountable.　(3/each; total: 15)

6. Consider the following conditional statement:
   　　　P: If the flood destroys my house or the fire destroys my house, then my insurance company will pay me.
   (a) Which of the following is the converse of P?　(3/each; total: 6)
   (b) Which of the following is the contrapositive of P?
   　　　Q: If my insurance company pays me, then the flood destroys my house or the fire destroys my house.
   　　　R: If my insurance company pays me, then the flood destroys my house and the fire destroys my house.
   　　　S: If my insurance company does not pay me, then the flood does not destroy my house or the fire does not destroy my house.
   　　　T: If my insurance company does not pay me, then the flood does not destroy my house and the fire does not destroy my house.

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7. What is the negation of the statement "2 is even or –3 is negative"?   (total: 4)

8. Find all sublattices of D_{24} that contain at least five elements.   (total: 4)

9. Let $S = \{1, 2, 3, 4, 5\}$ and let $A = S \times S$. define the following relation R on A: (a, b) R (c, d) if and only if $ad = cb$.   (a: 12, b: 8; total: 20)
   (a) Show that R is an equivalence relation.
   (b) Compute A/R.