| 科目 | 離散數學 | 適用系所 | 資訊工程學系 | 時間 | 一〇〇分鐘 |
|------|----------|----------|--------------|------|-----------|

※請務必在答案卷作答區內作答。　　　　　（共三頁）

**1. (10%)** If $n \in Z^+$, prove that 43 divides $6^{n+2} + 7^{2n+1}$.

**2. (10%)** Determine whether each of the following statements is true or false. If true, briefly explain the reason. If false, provide a counterexample. The universe comprises all integers.

(a) $\forall x \exists y \exists z (x = 3y + 5z)$

(b) $\forall x \exists y \exists z (x = 3y + 6z)$

**3. (15%)** Find the value of bb after the given program segment is executed. (Here i, j, k, aa and bb are integer variables.)

```
aa = 0;

bb = 1;

for ( i = 1; i <= 30; i + + )

    for ( j = 1; j <= i; j + + )

        for ( k = 1; k <= j; k + + )

        {

                if ( k = = j )

                {

                    aa = aa + 1;

                    bb = bb * aa;

                }

        }
```

**4.** (15%) For the finite state machine ($M$) given in the following table, determine a minimal machine that is equivalent to it, where $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$ the internal states for $M$; $I = \{0,1\}$ the input alphabet; $O = \{0,1\}$ the output alphabet; $\nu$ the next function; and $w$ the output function.

|       | $\nu$ | | $w$ | |
|-------|-----|-----|-----|-----|
|       | 0   | 1   | 0   | 1   |
| $s_0$ | $s_1$ | $s_2$ | 1 | 0 |
| $s_1$ | $s_2$ | $s_3$ | 0 | 1 |
| $s_2$ | $s_3$ | $s_1$ | 1 | 0 |
| $s_3$ | $s_5$ | $s_7$ | 1 | 1 |
| $s_4$ | $s_4$ | $s_0$ | 0 | 1 |
| $s_5$ | $s_3$ | $s_6$ | 1 | 0 |
| $s_6$ | $s_5$ | $s_3$ | 0 | 1 |
| $s_7$ | $s_6$ | $s_7$ | 0 | 0 |

**5.** (10%) A Double Tower of Hanoi contains $2n$ disks of $n$ different sizes, two of each size. As usual, we're required to move only one disk at a time, without putting a larger one over a smaller one. How many moves does it take to transfer a double tower from one peg to another, if disks of equal size are indistinguishable from each other?

**6.** (40%) **Choose the best answer. Each 4%.**

(1) Let S = {1, 2, 3, ..., 35}. How many elements must we select from S to insure that there will be at least two whose sum is 36?
(a)17　(b) 18　(c)19　(d)20　(e) none of the above.

(2) If $|A| = 30$ and the equivalence relation $R$ on $A$ partitions $A$ into equivalence classes $A_1, A_2$ and $A_3$, where $|A_1| = |A_2| = |A_3|$, what is $|R|$?

(a) 30　(b) 300　(c) 100　(d) 60　(e) none of the above.

(3) How many integer solutions are there to $x_1 + x_2 + x_3 + x_4 = 18$, where $0 \le x_i \le 7, \forall 1 \le i \le 4$?
(a) 150　(b) 160　(c) 180　(d) 246　(e) none of the above

(4) What is the coefficient of $x^3$ in $(1-4x)^{-1/2}$?

(a) 20　(b) 8　(c) 32　(d) 16　(e) none of the above.

(5) Seventeen students plan to have dinner together for several days. They will be seated at a round table, and the plan calls for each student to have different neighbors at every dinner. For how many days at least can this be done?
(a) 11　(b) 13　(c) 15　(d) 17　(e) none of the above

**(6)** Which of the following statements are true?

    (a) Any subgraph of a bipartite graph is bipartite.

    (b) Every loop-free connected planar graph has a vertex $v$ with deg $(v) < 6$.

    (c) Let $G = (V, E)$ be a loop-free connected planar graph with $|V| = v$, $|E| = e$, and $r$ regions. Then $e \leq 3v - 6$.

    (d) all of the above
    (e) none of the above

**(7)** If a tree has four vertices of degree 2, one vertex og degree 3, two vertices of degree 4, and one of degree 5, how many pendant vertices (leaves) does it have?

    (a) 10    (b)11    (c) 12    (d) 9    (e) none of the above.

**(8)** Construct an optimal prefix code for the symbols $a, o, q, u, y, z$ that occur (in a given sample) with frequencies 20, 28, 4, 17, 12, 7, respectively. What is the length of the codeword for the symbol $z$ ?

    (a) 2    (b) 3    (c) 4    (d) 5    (e) none of the above.

**(9)** What is $[25]^{-1}$ in $Z_{72}$ ?

    (a) 20    (b) 25    (c) 47    (d) 49    (e) none of the above.

**(10)** For $\Sigma = \{0,1\}$, the string 00010 is in which of the following languages ?

    (i) $\{0,1\}^*$    (ii) $\{0\}^*\{1\}^*\{0\}^*$    (iii) $\{00\}\{0\}^*\{10\}$    (iv) $\{00\}^*\{10\}^*$

    (a) only (i) and (ii)      (b) only (i) and (iii)      (c) only (ii) and (iv)
    (d) only (i), (ii) and (iii)      (e) none of the above