逢甲大學95學年度碩士班招生考試試題

科目	離散數學	適用系所	資訊工程學系	時間	100 分鐘

※請務必在答案卷作答區內作答。

1. (10%) Use the pigeonhole principle to show that if 101 integers are selected from the set $S = \{1, 2, 3, \cdots, 200\}$, then there are two integers such that one divides other.

2. (10%) Let f, g, and h be functions from N to N, where N is the set of natural numbers so that $f(n) = n + 1$, $g(n) = 2n$, and

$$h(n) = \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd} \end{cases}$$

Determine composition functions: (a) $f \circ g$, (b) $g \circ g$, (c) $g \circ h$, (d) $h \circ g$, and (e) $(f \circ g) \circ h$.

3. (10%) Evaluate the following:

(a) $\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2$.

(b) $\binom{n}{1} + 2\binom{n}{2} + \cdots + n\binom{n}{n}$.

4. (10%) If $A = \{1, 2, 3, 4\}$, determine the following:
(a) how many binary relations on A?
(b) how many equivalence relations on A?

5. (10%) Let π_1 and π_2 be two partitions of a set A, and let R_1 and R_2 be the corresponding equivalence relations. We define the product of π_1 and π_2, denoted by $\pi_2 \cdot \pi_2$, to be the partition corresponding to the equivalence relations $R_1 \cap R_2$. We define the sum of π_1 and π_2, denoted by $\pi_2 + \pi_2$, to be the partition corresponding to the equivalence relations $(R_1 \cup R_2)^*$ (transitive closure of $R_1 \cup R_2$). Now, let $A = \{1, 2, 3, \cdots, 11\}$, $\pi_1 = \{\{1, 2, 3, 4\}, \{5, 6, 7\}, \{8, 9\}, \{10, 11\}\}$ and $\pi_2 = \{\{1, 2, 3, 8\}, \{4, 9\}, \{5, 6, 10, 11\}, \{7\}\}$. Find $\pi_2 \cdot \pi_2$ and $\pi_2 + \pi_2$.

6. (10%) For all $n \in Z$, $n \geq 0$, prove that $n^3 + (n + 1)^3 + (n + 2)^3$ is divisible by 9.

7. (10%) Determine whether each of the following statements is true or false. If true, briefly explain the reason. If false, provide a counterexample. The universe comprises all integers.

(a) $\forall x\, \exists y\, \exists z\, (x = 6y + 7z)$

(b) $\forall x\, \exists y\, \exists z\, (x = 6y + 8z)$

8. (15%) One hundred students enter a locker room that contains 100 lockers. The first student opens all the lockers. The second student changes the status (from closed to open, and vice versa) of every other locker, starting with the second locker. The third student then changes the status of every third locker, starting with the third locker. In general, for $1 \leq k \leq 100$, the kth student changes the status of every kth locker, starting with the kth locker. After the 100th student has gone through the lockers, which lockers are left open?

9. (15%) Find the value of bb after the following program segment is executed. (Here i, j, k, aa and bb are integer variables.)

```
aa = 0;
bb = 1;
for(i=1;i<=20;i++)
   for(j=1;j<=10;j++)
      for(k=1;k<=j;k++)
      {
            if (j == i)
            {
               aa = aa + 1;
               bb = bb * aa;
            }
      }
```