

逢甲大學96學年度碩士班招生考試試題

編號：071

科目	離散數學	適用系所	生醫資訊暨生醫工程碩士學位學程、資訊工程系	時間	100 分鐘
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※請務必在答案卷作答區內作答。

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1. (40%) Choose the best answer. Each 4%.

(1) Consider the following program segment, where i , j , and k are integer variables.

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for i:= 1 to 20 do
  for j:= 1 to i do
    for k:= 1 to j do
      print (i×j+k)
    
```

How many times is the print statement executed in this program segment?

- (a) 20^3 (b) $\binom{22}{3}$ (c) $\binom{20}{3}$ (d) $\binom{20}{4}$ (e) none of the above.

(2) How many of the following statements are true?

- (i) $\exists x[p(x) \vee q(x)] \Leftrightarrow [\exists x p(x) \vee \exists x q(x)]$
(ii) $\forall x[p(x) \wedge q(x)] \Leftrightarrow [\forall x p(x) \wedge \forall x q(x)]$
(iii) $[(p \rightarrow q) \wedge (\neg r \vee s) \wedge (p \vee r)] \rightarrow (\neg q \rightarrow s)$
(iv) $[p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)] \rightarrow (s \vee t)$
(v) $p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r) \Leftrightarrow p \vee \neg r$

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5.

(3) How many total ordering relations are there on a set with 5 distinct elements?

- (a) 120 (b) 2^{10} (c) 52 (d) 203 (e) none of the above.

(4) In how many ways can one travel in the xy -plane from $(2, 1)$ to $(7, 6)$ using the moves $R: (x, y) \rightarrow (x+1, y)$ and $U: (x, y) \rightarrow (x, y+1)$, if the path taken may touch but never rise above the line $y = x - 1$?

- (a) 14 (b) 32 (c) 42 (d) 132 (e) none of the above.

(5) Let $A = \{1, \{1\}, \{2\}\}$. How many of the following statements are true?

- (i) $1 \in A$ (ii) $\{1\} \in A$ (iii) $\{\{1\}\} \subseteq A$ (iv) $\{2\} \in A$ (v) $\{2\} \subseteq A$

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5.

(6) Let $n \in \mathbb{Z}^+$. How many of the following statements are true?

(i) $\gcd(8n+3, 5n+2) = 1$.

(ii) $\gcd(n, n+2) = 2$.

(iii) Let $a, b, p \in \mathbb{Z}^+$. If $p \mid ab$, then $p \mid a$ or $p \mid b$.

(iv) $3 \mid 7^n - 4^n$

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4.

(7) Consider the following finite state machine.

	v		w	
	0	1	0	1
s_1	s_6	s_3	0	0
s_2	s_3	s_1	0	0
s_3	s_2	s_4	0	0
s_4	s_7	s_4	0	0
s_5	s_6	s_7	0	0
s_6	s_5	s_2	1	0
s_7	s_4	s_1	1	1

Which of the following statements are true?

(a) s_2 and s_7 are equivalent.

(b) s_1 and s_5 are equivalent.

(c) s_2 and s_5 are equivalent.

(d) all of the above.

(e) none of the above.

(8) For $\Sigma = \{0,1\}$, the string 00010 is in which of the following languages ?

(i) $\{0,1\}^*$ (ii) $\{000,101\}\{10,11\}$ (iii) $\{00\}\{0\}^*\{11\}\{0\}^*$ (iv) $\{00\}^*\{11\}^*$

(a) only (i) and (iii)

(b) only (iv)

(c) (i), (ii) and (iii)

(d) only (ii) and (iii)

(e) none of the above

(9) A ship carries 48 flags, 12 each of the colors red, white, blue, and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. How many of these signals use an even number of blue flags and an odd number of black flags?

(a) 4^{12}

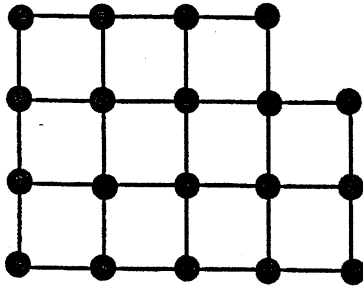
(b) 4^{11}

(c) $12!$

(d) $48!/12!$

(e) none of the above.

(10) Consider the following graph G.



How many of the following statements are true?

- (i) G has an Euler circuit. (ii) G has a Hamilton path.
 (iii) The chromatic number of G is 2.
 (iv) G is a bipartite graph.

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4.

2. (10%) Find a simultaneous solution for the system of four congruences:

$$x \equiv 1 \pmod{2}$$

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{5}$$

$$x \equiv 5 \pmod{7}$$

3. (10%) Using binomial expansions to evaluate:

(a) $\sum_{k=0}^n \binom{n}{k} 2^k$ (b) $\sum_{k=0}^n k \binom{n}{k}^2$

4. (10%) In how many ways can integers 1, 2, 3, 4, 5, 6, 7, 8 and 9 be permuted such that no odd integer will be in its natural position?

5. (10%) Among the 4^n n-digit quaternary sequences, how many of them have an even number of 0's?

6. (10%)

- (a) Given a group of n women and their husbands, how many people must be chosen from this group of 2n people to guarantee the set contains a married couple?
- (b) Show that at any party with at least six people, there either exists a set of three mutual friends or a set of three mutual strangers.

7. (10%) Let a, b, c be positive integers, prove that the Diophantine equation $ax + by = c$ has an integer solution $x = x_0, y = y_0$ iff $\gcd(a, b)$ divides c .