逢甲大學97學年度碩士班招生考試試題　編號：067

科目	離散數學	適用系所	資訊工程學系	時間	100 分鐘

※請務必在答案卷作答區內作答。

1. (20%) Choose the best answer.

(1) In how many ways can one go from (0,0) to (7,3) if the only moves permitted are R: (x, y)→(x+1, y) and U: (x, y)→(x, y+1), and the number of U's may never exceed the number of R's along the path taken?

(a) 75 **(b)** 120 **(c)** 10! **(d)** 3!7! **(e)** none of the above

(2) How many positive integers n divide $100137n + 248396544$?

(a) 48 **(b)**126 **(c)**144 **(d)**252 **(e)** none of the above

(3) A prefix code for {a, b, c, d, e, f} is given by a: 00, b: 01, c:101, d:x10, e:yz1, f: 10w1, where x, y, z, w∈ {0, 1}. Then x + y+ z + w =

(a) 0 **(b)**1 **(c)**2 **(d)**3 **(e)** 4

(4) If A = {1, 2, 3, 4, 5}, how many equivalence relations on A are there?

(a) 15 **(b)** 25 **(c)** 51 **(d)** 203 **(e)** none of the above

(5) The n - dimension hypercube Q_n is an undirected graph with 2^n vertices which are labeled with the 2^n n - digit binary numbers. There is an edge between two vertices if their binary labels differ exactly at one digit. How many of the following statements are true?

(i) Q_n is bipartite .

(ii) Q_n has a Hamilton cycle.

(iii) Q_n has an Euler circuit.

(iv) The diameter of Q_n is n.

(a) 4 **(b)** 3 **(c)** 2 **(d)** 1 **(e)** 0

2. (10%) Find the value of *sum* after the given program segment is executed.(Here i, j, k, *increment*, and *sum* are integer variables.)

```
increment := 0
sum := 0
for i := 1 to 10 do
  for j := 1 to i do
    for k := 1 to j do
      begin
        increment := increment + 1
        sum := sum + increment
      end
```

3. (10%) Let P(X) denote the power set of a set X, i.e. P(X) = the set of all the subset of X. Prove or disprove each of the following for sets $A, B \subseteq U$

(1) $P(A \cup B) = P(A) \cup P(B)$

(2) $P(A \cap B) = P(A) \cap P(B)$

4. (10%) A proper k-edge-coloring of G is an assignment of k colors to the edges of G such that no two adjacent edges have the same color. The edge chromatic number of a graph G, denoted by $\chi'(G)$, is the minimum k for which G has a proper k-edge-coloring. Let K_n be the complete graph of n vertices. What is the value of $\chi'(K_n)$? Explain.

5. (10%) Let A={1, 2, 3, 4, 5} and B={1, 2, 3}

(1) How many functions f: A→B are there?

(2) How many functions f: A→B are one-to-one?

(3) How many functions f: A→B are onto?

(4) How many functions f: A→B are nondecreasing?

(5) How many functions g: B→A are one-to-one?

6. (10%) Use a combinatorial argument to prove that

$$\frac{(3n)!}{2^n \times 3^n} \text{ and } \frac{(n^2)!}{(n!)^{n+1}} \text{ are integers.}$$

7. (1) (5%) Let $x_1, x_2, ..., x_n$ be arbitrary integers. Show that $x_i + x_{i+1} + ... + x_{i+k}$ is divisible by n for some $i \geq 1$ and $k \geq 0$.

(2) (10%) Use the pigeonhole principle to show that the decimal expansion of a rational numbers must, after some point, become periodic.

8. (1) (5%) Construct a finite-state machine M_1 whose output tells the number of finite symbols, modulo 3, that have been applied.

(2) (10%) The input and output alphabets of machine M_2 are {0, 1}. Construct a finite-state machine M_2 whose output sequence r(t)r(t-1)...r(1) is to be a replica of the input sequence s(t)s(t-1)...s(1) delayed by two units:

r(t) = s(t-2) for t>2.

We do not care what r(1) and r(2) are.