

科目	離散數學	適用系所	資訊工程學系	時間	100 分鐘
----	------	------	--------	----	--------

※請務必在答案卷作答區內作答。

第 1 頁 共 2 頁

1. (20%) Choose the best answer.

(1) Let $T = (V, E)$ be a tree with $|V| = 10$. How many distinct paths are there (as subgraphs) in T ?

- (a) 25 (b) 35 (c) 45 (d) 50 (e) none of the above

(2) Which of the following statements are true?

- (a) $\exists x[p(x) \vee q(x)] \Leftrightarrow [\exists xp(x) \vee \exists xq(x)]$
 (b) $\forall x[p(x) \wedge q(x)] \Leftrightarrow [\forall xp(x) \wedge \forall xq(x)]$
 (c) $[(p \rightarrow q) \wedge (\neg r \vee s) \wedge (p \vee r)] \rightarrow (\neg q \rightarrow s)$
 (d) all of (a), (b), (c)
 (e) none of the above

(3) Eleven students plan to have dinner together for several days. They will be seated at a round table, and the plan calls for each student to have different neighbors at every dinner. For how many days at least can this be done?

- (a) 5 (b) 7 (c) 9 (d) 11 (e) none of the above

(4) Let $\phi(n)$ be the number of positive integers m , where $1 \leq m < n$ and $\gcd(m, n) = 1$, e.g. $\phi(4) = 2, \phi(5) = 4$. Find $\phi(420) = ?$

- (a) 32 (b) 64 (c) 96 (d) 105 (e) none of the above

(5) If $\sum_{i=0}^{25} \binom{25}{i} 8^i = x^{50}$, then $x = ?$

- (a) 4 (b) 3 (c) ± 4 (d) ± 3 (e) none of the above

2. (10%) Determine the number of integer solutions for $x_1 + x_2 + x_3 + x_4 = 18$, where

- (1) $x_1, x_2, x_3 > 0, x_4 \geq 5$.
 (2) $0 \leq x_i \leq 7, \forall 1 \leq i \leq 4$.

3. (10%) Prove or disprove the following statement:

For all $n \in \mathbb{Z}^+$ and $n \geq 24$, n can be written as a sum of 5's and/or 7's.

4. (10%) In how many ways can 30030 be factored into three factors, each greater than 1, if the order of the factors is not relevant?

5. (5%) Solve the system of congruence equations

$$\begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 2 \pmod{5} \\ x \equiv 3 \pmod{7} \end{cases}$$

6. (10%) Let A be the set of positive integer divisors of 1800. We define the relation R on A by xRy if x divides y . How many ordered pairs are there in the relation R ?

7. Suppose $A = \{1, 2, 3, 4, 5\}$.

(1) (10%) How many equivalence relations on A are there?

(2) (5%) If an equivalence relation R on A induces the partition $A = \{1, 2\} \cup \{3\} \cup \{4, 5\}$. What is R ?

8. (10%) Consider the $n \times n$ determinant D_n given by

$$\begin{vmatrix} 2 & 2 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 2 & \dots & 0 & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \dots & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \dots & 2 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 2 & 2 \end{vmatrix}$$

Find the value of D_n as a function of n .

9. (10%) How many rooted ordered trees on n vertices?