第一題組

1. (10%) You are the lead designer of a new processor. The processor design and compiler are complete, and now you must decide whether to produce the current design as it stands or spend additional time to improve it.

You discuss this problem with your hardware engineering team and arrive at the following options:

a. *Leave the design as it stands.* Call this base machine *Mbase.* It has a clock rate of 500 MHz, and the following measurements have been made using a simulator:

Instruction class	CPI	Frequency
A	2	40%
B	3	25%
C	3	25%
D	5	10%

b. *Optimize the hardware.* The hardware team claims that it can improve the processor design to give it a clock rate of 600 MHz. Call this machine *Mopt.* The following measurements were made using a simulator for Mopt:

Instruction class	CPI	Frequency
A	2	40%
B	2	25%
C	3	25%
D	4	10%

What is the CPI for each machine?

2. (10%)
How much faster is Mopt than Mbase in Problem 1 ?

3 (5%) The compiler team has heard about the discussion to enhance the machine discussed in Problem 1 and 2. The compiler team proposes to improve the compiler for the machine to further enhance performance. Call this combination of the improved compiler and the base machine *Mcomp*. The instruction improvements from this enhanced compiler have been estimated as follows:

Instruction class	Percentage of instructions executed vs. base machine
A	90%
B	90%
C	85%
D	95%

For example, if the base machine executed 500 class A instructions, Mcomp would execute $0.9 \times 500 = 450$ class A instructions for the same program. What is the CPI for Mcomp?

4. (5%) Using the data of Problem 1, how much faster is Mcomp than Mbase?

5 (10%) The compiler group points out that it is possible to implement both the hardware improvements of Problem 1 and the compiler enhancements described in Problem 3. If *both* the hardware and compiler improvements are implemented, yielding machine *Mboth*, how much faster is Mboth than Mbase?

6 (10%) You must decide whether to incorporate the hardware enhancements suggested in Problem 1, or the compiler enhancements of Problem 3 (or both) to the base machine described in Problem 1. You estimate that the following time would be required to implement the optimizations described in Problem 1, 3, and 5:

Optimization	Time to implement	Machine name
Hardware	6 months	Mopt
Compiler	6 months	Mcomp
Both	8 months	Mboth

Recall that CPU performance improves by approximately 50% per year, or about 3.4% per month. Assuming that the base machine has performance equal to that of its competitors, which optimizations (if any) would you choose to implement?

第二題組：

(50%) **Floating-Point Arithmetic**

The general expression for a floating-point number is

$$(-1)^S \times (1 + Significand) \times 2^{(Exponent-Bias)}.$$

According to the standard of IEEE 754, a single-precision floating-point representation contains 1 sign bit, 8 exponent bits, 23 significand bits, and the bias is 127. A double precision number has 1 sign bit, 11 exponent bits, 52 significand bits, and the bias is 1023.

7. (5%) What is the range of single-precision numbers? In other words, what is the largest positive number and what is the smallest negative number in single-precision representation? Compare the range with the 32-bit 2's complement fixed-point representation?

8. (5%) What is the range of double-precision numbers? Compare the range with the 64-bit 2's complement fixed-point representation?

9. (5%) Show the binary representation of the decimal number of −0.625 in both single and double precision.

10. (5%) Show how computers calculate the floating-point addition of two single-precision numbers with decimal values of 0.75 and 128. Is the floating-point addition faster or slower than the fixed-point addition? Why?

11. (20%) Draw a block diagram of an arithmetic unit dedicated to floating-point addition of double-precision. It should contains blocks of ALUs, shifters, multiplexers, increment units, decrement units, rounding hardware, registers and other control units. Explain clearly the operation of each block in the block diagram and the operation of the floating-point addition using a flow-chart (algorithm).

12. (10%) In Pentium CPU, there are two integer arithmetic units and a floating-point unit. Explain the difference of functions of these units. Does the Pentium CPU use ALUs to calculate the floating-point multiplication? Describe how the Pentium CPU performs the arithmetic operations of addition, subtraction, multiplication, and division using the available hardware inside the microprocessor chip.

1. (10%) A microcomputer uses the buddy system for memory management. Initially, it has one block of 256K at address 0. After successive requests for 5K, 25K, 35K, and 20 K come in, how many blocks are left and what are their sizes and addresses ?.
2. (10%) Five jobs are waiting to be run. Their expected run times are 9, 6, 3, 5 and X. In what order should they be run to minimize average response time ? (Your answer will depend on X.)
3. (10%) A computer system has a main memory consisting of 32K 16-bit words. it also has a 4K-word cache organized in the set-associative memory with four blocks per set and 64 words per block. Assume that the cache is initially empty. Suppose that the CPU fetches 4352 (= 272 blocks) words from location 0, 1, 2, ..., 4351, in order. It then repeats this fetch sequence nine more times. The cache is 10 times faster than the main memory. Estimate the improvement factor resulting from the use of the cache. Assume that the LRU algorithm is used for block replacement.
4. (10%) Suppose that a moving-head disk contains 200 tracks (numbered through 199) and that the head is currently serving a request at track 43 and has just finished a request at track 126. We have the following requests in the FIFO order:
   86, 147, 91, 177, 94, 150, 102, 175, 130.
   What is the total number of head movement needed to satisfy these requests for the following disk-scheduling algorithms ?
   (a) FCFS.
   (b) SSTF.
   (c) SCAN.
   (d) LOOK.
   (e) C-SCAN.
5. (10%) Which of the following disk scheduling algorithms
   (a) has the highest variance of response time ?
   (b) has the lowest variance of response time ?
   (c) has the best throughput ?
   (d) has the poorest throughput ?
   (e) is the most unacceptable scheduling algorithm in an interactive system ?
   FCFS, SSTF, LOOK, SCAN, CSCAN.

6. (10%) If we do not regard two trees as different when they differ in only the respective ordering of subtrees of nodes, the tree is said to be oriented.

(a) How many different oriented trees are there with four nodes A, B, C, and D?

(b) How many different oriented trees are there with n nodes?

7. (10%) When we traverse a tree, we find the tree in preorder is ABDCEGFHJ, the tree in inorder is DBAEGCHFJ, and the tree in postorder is DBGEHJFCA.Please draw the tree.

8. (10%) If a tree has a node of degree 1, two nodes of degree two, …, n nodes of degree n, how many terminal nodes are there in the tree?

9. (10%) With the node structure as shown as shown below.

exp	right
cv	down

In the node structure, the right and down are links, the exp is an integer representing an exponent, and the cv is either a constant (coefficient) or the alphabetic name of a variable. What is the computer representation for $5 + x^4 + 2z^5 - 12x^2z^5$?

10. (10%) (a) The degree d(v) of a vertex v in a graph is the number of edges of the graph incident with v, then can $\sum_{v \in vertex} d(v)$ be represented by total number of edges? How?

(b) If a graph is connected, then the number of edges is greater than or equal to (1) the number of vertices, (2) the number of vertices minus one, or (3) the number of vertices plus one.

(c) Every nontrivial tree has at least how many of vertices with degree one?

國立中山大學八十八學年度碩博士班招生考試試題

科　目：離散數學（資工所）

1. The following terms are used in graph theory. Explain each of them. Give an example for each term. (20%)
   (a) bipartite graph
   (b) connected component
   (c) tree
   (d) Euler path
   (e) transitive closure

2. Consider the equation $x_1 + x_2 + x_3 + x_4 = 7$, where each x_i is a nonnegative integer. How many distinct solutions are there? Note that $(x_1, x_2, x_3, x_4) = (2, 3, 1, 1)$ and $(x_1, x_2, x_3, x_4) = (3, 2, 1, 1)$, for example, are considered to be distinct. (10%)

3. In the classroom, the teacher and students play a guessing game. There is one gift put in one of three boxes, labelled as A, B, C. The three boxes are closed, so that no student, except the teacher, knows which box contains the gift. Suppose the three boxes are fair and the probability of each one containing the gift is $\frac{1}{3}$. Now, you are the student to guess which box the gift is in. If you give the correct answer, you will get the gift and be happy; otherwise you will be unhappy. Suppose you guess the gift is in box B. The teacher privately checks the other two boxes, A and C (You still do not see the content of A and C.). Then, the teacher open one empty box, chosen from A and C (There must be at least one empty box in A and C.). Therefore, you now know one empty box, which may be A or C. And the teacher give you another chance to change your selection, that is, you can still select B or change your mind to select the other one (not the empty one). Will you change your mind? What are the probabilities of B and the other one getting the gift, respectively? Give your reason. (15%)

4. Let $b_n, n \geq 0$ be a sequence of integers satisfying the third-order inhomogeneous difference equation

$$\begin{aligned} b_0 = b_1 &= 0 \\ b_2 &= 2 \\ b_n - 2b_{n-1} - b_{n-2} + 2b_{n-3} &= 1 \quad if \quad n = 3 \\ b_n - 2b_{n-1} - b_{n-2} + 2b_{n-3} &= 0 \quad if \quad n \geq 4. \end{aligned}$$

   Solve the equation. (20%)

5. The block cipher is an encryption method based upon Chinese Remainder Theory. In the method, we are given t data keys, $m_1, m_2, \cdots, m_t$. In the enciphering phase, the t data keys (plain text) are encoded with t encipher keys, and a single enciphered value (cipher text) C is obtained. While in the deciphering phase, C is decoded with $d_1, d_2, \cdots, d_t$, then the original data keys (plain text) $m_1, m_2, \cdots, m_t$ are restored. Suppose $d_1, d_2, \cdots, d_t$ are defined first. Let $n = d_1 d_2 \cdots d_t$. Then $e_j, 1 \leq j \leq t$ is calculated by the following way:

$$e_j = (\frac{n}{d_j}) y_j \bmod n$$

   where

$$(\frac{n}{d_j}) y_j \equiv 1 (\bmod d_j)$$

   C is calculated as follows:

$$C = \left(\sum_{j=1}^{t} e_j m_j \right) \bmod n$$

(a) Let $t = 3, (m_1, m_2, m_3) = (2, 1, 3)$. And suppose $d_1 = 3, d_2 = 4, d_3 = 5$. What are the values of e_1, e_2, e_3 ? (6%)

(b) How do you calculate m_1, m_2, m_3 from C with d_1, d_2, d_3? Prove your answer. (9%)

6. Let $S_n = \{1, 2, \cdots, n\}$. An m-permutation of S_n is obtained by selecting m distinct integers out of the n integers and arranging them in some order. Now let $x = (x_1\ x_2\ \cdots\ x_m)$ and $y = (y_1\ y_2\ \cdots\ y_m)$ be two m-permutations of S_n. We say that x precedes y in *lexicographical order* if there exists an i, $1 \le i \le m$, such that $x_j = y_j$ for all $j < i$ and $x_i < y_i$. And the *rank* of an m-permutation p, denoted as $r(p)$, is its position in the lexicographical order of all m-permutations. For example, all 3-permutations of S_5 in lexicographical order are (1 2 3), (1 2 4), (1 2 5), (1 3 2), (1 3 4), (1 3 5), (1 4 2), (1 4 3), (1 4 5), (2 1 3), $\cdots$, (5 4 2), (5 4 3). And $r(1\ 2\ 3) = 1$, $r(1\ 3\ 5) = 6$, $r(5\ 4\ 3) = 60$ and etc. Answer the following questions for the 4-permutations of S_7.

(a) What is the next one of (4 1 6 7)? (3%)

(b) Which permutation has the rank 835? Explain how you calculate. (5%)

(c) What is $r(4\ 1\ 6\ 7)$? Explain how you calculate. (6%)

(d) Which permutation has the rank 425? Explain how you calculate. (6%)