If some questions are unclear or not well defined to you, you can make your own assumptions and state them clearly in the answer sheet.

1. Short Questions (25%)

Answer and explain the following questions. Credit will be given only if explanation is provided.

1.1. Explain the problems with using MIPS (million instructions per second) as a measure for comparing machines.

1.2 There are two possible improvements to enhance a machine: either make multiply instructions run four times faster than before, or make memory access instructions run two times faster than before. You repeatedly run a program that takes 100 seconds to execute. Of this time, 10% is used for multiplication, 50% for memory access instructions, and 40% for other tasks. What will the speedup be if you improve only memory access? What will the speedup be if both improvements are made?

1.3 Explain and compare microprogrammed control and hardwired control.

1.4 There are two basic options when writing to the cache: write through and write back. Please explain write through and write back, and describe their advantages.

1.5 Explain the differences among superpipelining, superscalar and dynamic pipeline scheduling.

2. Performance Analysis (16%)

2.1 (4%) The PowerPC, made by IBM and Motorola and used in the Apple Macintosh, shares many similarities to MIPS. The primary difference is two more addressing modes (indexed addressing and update addressing) plus a few operations. Please explain the two addressing modes provided by PowerPC.

2.2 (6%) Consider an architecture that is similar to MIPS except that it supports update addressing for data transfer instructions. If we run gcc using this architecture, some percentage of the data transfer instructions shown in Fig. 1 will be able to make use of the new instructions, and for each instruction changed, one arithmetic instruction can be eliminated. If 20% of the data transfer instructions can be changed, which will be faster for gcc, the modified MIPS architecture or the unmodified architecture? How much faster? (Assume that the modified architecture has its cycle time increased by 10% in order to accommodate the new instructions.)

2.3 (6%) When designing memory systems, it becomes useful to know the frequency of memory reads versus writes as well as the frequency of accesses for instructions versus data. Assume that two-thirds of data transfers are loads. Using the average instruction-mix information for MIPS for the program gcc in Fig. 1, find the following:

(1) The percentage of all memory accesses that are for data (vs. instructions).

(2) The percentage of all memory accesses that are writes (vs. reads).

3. Computer Arithmetic (16%)

3.1 (5%) Suppose you wanted to add four numbers (A, B, E, F) using 1-bit full adders. There are two approaches to compute the sum as shown in Fig. 2(a) and 2(b): cascaded of traditional ripple carry adders and carry save adders. If A, B, E, F are 4-bit numbers, draw the detailed architecture (consists of 1-bit full adders) of the carry save addition shown in Fig. 2(b).

3.2 (3%) Assume that the time delay through each 1-bit full adder is 2T. Calculate and compare the times of adding four 8-bit numbers using the two different approaches.

3.3 (8%) Try Booth's algorithm for the signed multiplication of two numbers: $2_{ten} \times -3_{ten} = -6_{ten}$ (or $0010_{two} \times 1101_{two} = 1111\ 1010_{two}$). Explain the operations step by step.

4. Pipelining (19%)

4.1 (5%) MIPS instructions classically take five steps to execute in pipeline. Please explain the detailed operations of the five-stage pipeline used in MIPS instructions.

4.2 (10%) Fig. 3 shows a pipelined datapath of MIPS processor. Please explain the function of hazard detection unit and forwarding unit in Fig. 3 and they how to resolve data hazards.

4.3 (4%) Dynamic branch prediction is usually used to resolve control hazards. Consider a loop branch that branches nine times in a row, then is not taken once. What is the prediction accuracy for this branch when applying 1-bit and 2-bit prediction schemes respectively? (1-bit predictor updates the prediction bit on a mispredict, a prediction in 2-bit predictor must miss twice before it is changed)

5. Cache (24%)

5.1 (4%) How does the control unit deal with cache misses? Please describe the steps to be taken on an instruction cache miss as clear as possible.

5.2 (6%) Please explain the function of three portions (Tag, Index, and Block offset) in the address of Fig. 4. How many total bits are required for the cache?

5.3 (6%) Assume an instruction cache miss rate for gcc of 2% and a data cache miss rate of 4%. If a machine has a CPI of 2 without any memory stalls and the miss penalty is 50 cycles for all misses, determine how much faster a machine would run with a perfect cache that never missed. Use the instruction frequencies for gcc from Fig. 1.

5.4 (8%) Suppose we increase the performance of the machine in the previous question by doubling its clock rate. Since the main memory speed is unlikely to change, assume that the absolute time to handle a cache miss does not change. Assuming the same miss rate as the previous question, how much faster will the machine be with the faster clock?

Instruction class	MIPS example	Average CPI	Frequency	
			gcc	spice
Arithmetic	Add, sub, addi	1.0	48%	50%
Data transfer	lw, sw, lb, sb, lui	1.4	35%	41%
Conditional branch	beq, bne, slt, slti	1.7	15%	7%
Jump	j, jr, jal	1.2	2%	2%

Fig. 1

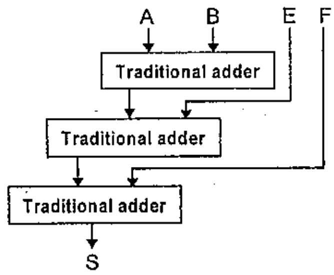


Fig. 2(a)

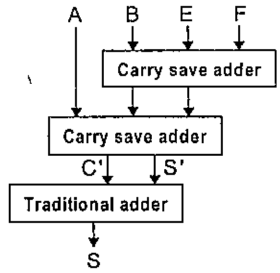


Fig. 2(b)

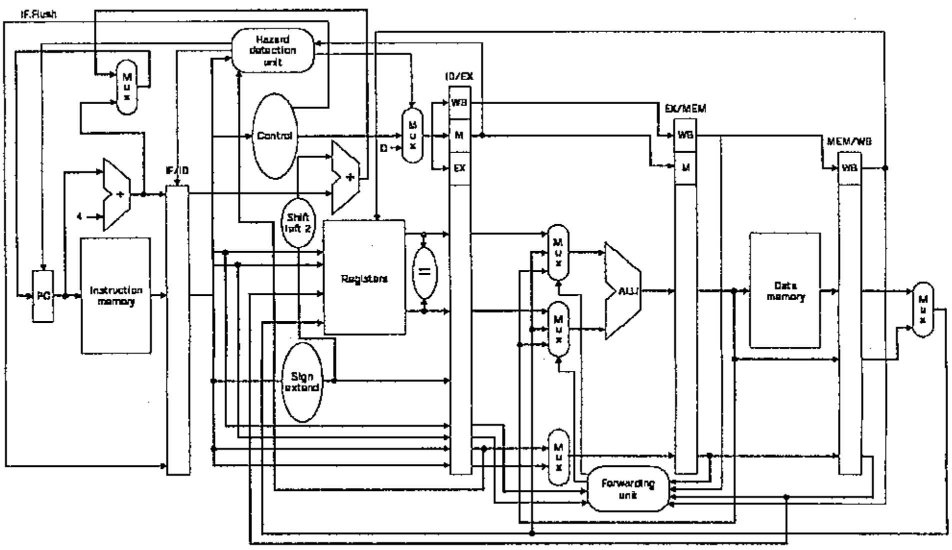


Fig. 3

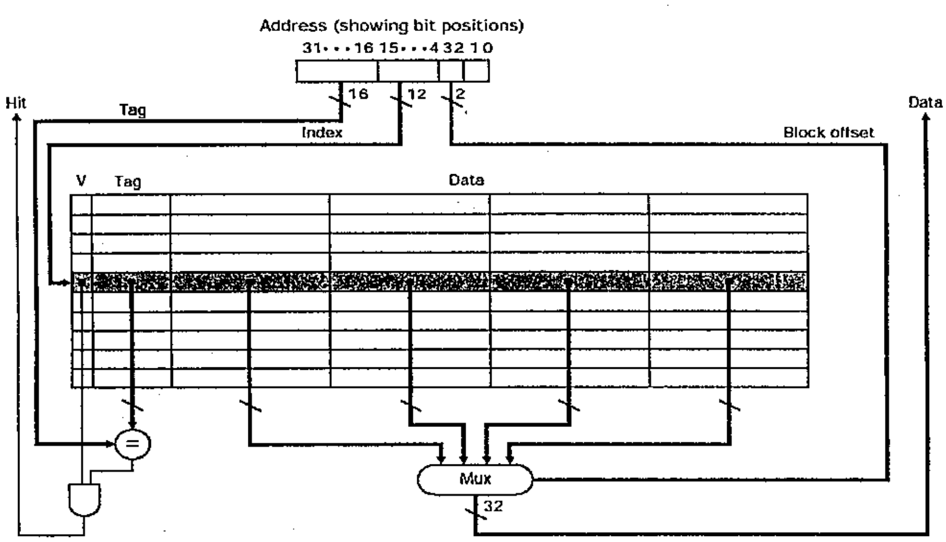


Fig. 4

國立中山大學九十四學年度碩士班招生考試試題

科目：離散數學【資工系碩士班甲組(含乙組選考)】

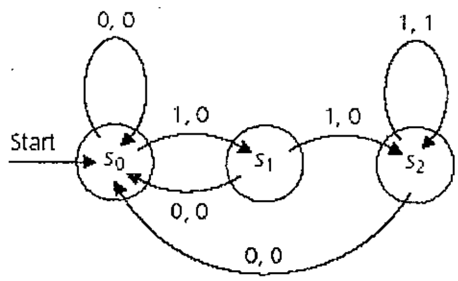
1. [10%] Find all of the possible solutions of

$$250x + 111y = 7$$

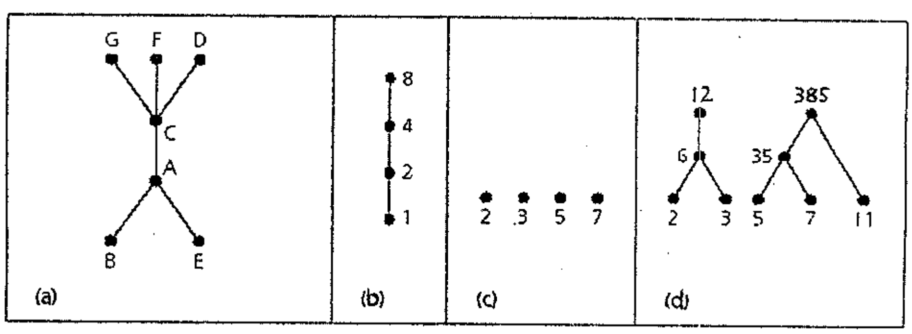
where both x and y are integers.

2. [10%] Let $A = \{0, 1, 2, 3, 4, 5, 6\}$ and $B = \{p, q, r, s\}$. How many onto functions are there from A to B?

3. [10%] Please describe the language that the following finite state machine can recognize.



4. Consider the following four Hasse diagrams:
(1) [5%] Which one (or ones) is (or are) totally ordered?
(2) [6%] Please write down the maximal elements of (c) and (d), respectively.
(3) [4%] Please write down the least elements of (a) and (c), respectively.



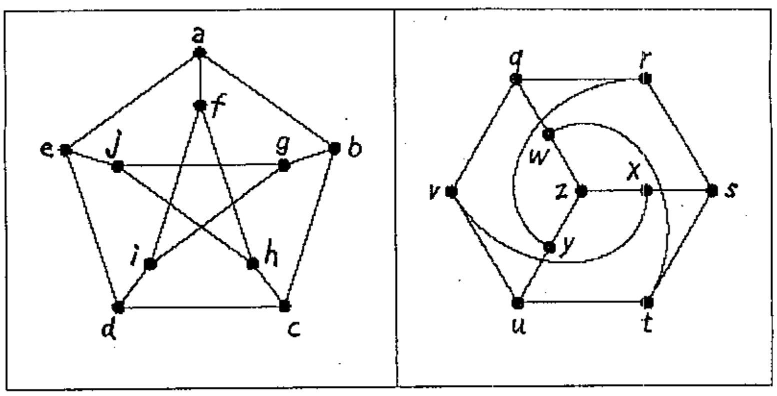
國立中山大學九十四學年度碩士班招生考試試題

科目：離散數學【資工系碩士班甲組(含乙組選考)】

5. [Generating Functions]
   (1) [5%] Please find the coefficient of x^5 in $\frac{1}{(1-2x)^7}$.
   (2) [10%] Determine the coefficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$.

6. [Recurrence Relations]
   (1) [6%] Solve $F_{n+2} = F_{n+1} + F_n$, where $n \geq 0$ and $F_0 = 0$, $F_1 = 1$.
   (2) [10%] Solve $a_{n+2} + 9a_n = 6a_{n+1} + 3(2^n) + 7(3^n)$, where $n \geq 0$ and $a_0 = 1$, $a_1 = 4$.

7. [10%] The following two graphs are isomorphic since there exists a function F : $\{a, b, c, d, e, f, g, h, i, j\} \rightarrow \{q, r, s, t, u, v, w, x, y, z\}$ such that F is a graph isomorphism. If $(F(a), F(b), F(e), F(f), F(i)) = (q, v, r, w, z)$, then $(F(c), F(d), F(g), F(h), F(j)) = ?$



8. [True/False Questions]
   (1) [2%] If $(F, +, \circ)$ is a field, then it must be an integral domain.
   (2) [2%] Any infinite integral domain $(D, +, \circ)$ is a field.
   (3) [2%] Let $n \in \mathbf{Z}^+$ and $n > 1$. $\mathbf{Z}_n$ is not a field if and only if n is composite.
   (4) [2%] Every group of prime order is acyclic.
   (5) [2%] If G is a finite group of order n with H a subgroup of order m, then $n|m$.
   (6) [2%] Congruence modulo n must be an equivalence relation on $\mathbf{Z}$.
   (7) [2%] If a graph contains a subgraph that is homeomorphic to K_5 or $K_{3,3}$, then the graph is not planar.

INSTRUCTIONS: *If any question is unclear or you believe some assumptions need to be made, state your assumptions clearly at the beginning of your answer.*

1. (5%) In a ring-protection system, level 0 has the greatest access to objects, and level n (greater than zero) has fewer access rights. The access rights of a program at a particular level in the ring structure are considered as a set of capabilities. What is the relationship between the capabilities of a domain at level j and a domain at level i to an object (for $j > i$)?

2. Assume a page reference string for a process with m frames (initially all empty). The page reference string has length p with n distinct page numbers occurring in it. For any page-replacement algorithms,
   (a) (5%) What is a lower bound on the number of page faults?
   (b) (5%) What is an upper bound on the number of page faults?

3. Consider a file currently consisting of 200 blocks. Assume that the file control block (and the index block, in the case of indexed allocation) is already in memory. Calculate how many disk I/O operations are required for contiguous, linked, and indexed (single-level) allocation strategies, if, for one block, the following conditions hold. In the contiguous allocation case, assume that there is no room to grow in the beginning, but there is room to grow in the end. Assume that the block information to be added is stored in memory.
   (a) (3%) The block is added at the beginning.
   (b) (3%) The block is added in the middle.
   (c) (3%) The block is added at the end.
   (d) (3%) The block is removed from the beginning.
   (e) (3%) The block is removed from the end.

4. (10%) Consider the interprocess-communication scheme where mailboxes are used. Suppose a process P wants to wait for two messages, one from mailbox A and one from mailbox B. What sequence of `send` and `receive` should it execute so that the messages can be received in any order?

5. Consider a system with four processes P_1 through P_4 and five allocatable resources R_1 through R_5. The current allocation and maximum needs are as follows:

	Allocated	*Maximum*	*Available*
P_1	1 0 2 1 1	1 1 5 1 2	0 0 x 1 1
P_2	2 0 1 1 0	5 2 2 1 0	
P_3	1 1 0 1 0	3 1 3 1 0	
P_4	1 1 1 1 0	1 1 2 2 1	

   (a) (5%) What is the smallest value of x for which this is a safe state?
   (b) (5%) What is the safe sequence?

6. (10%) Find a closed form solution for the recurrence relation $T(n) = T(\sqrt{n}) + \lg n$.

7. What is the worst-case running time of the following sorting algorithms?

   (a) (2%) BUBBLESORT

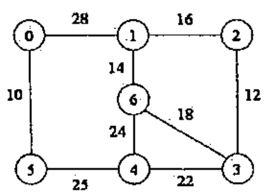
   (b) (2%) INSERTIONSORT

   (c) (2%) MERGESORT

   (d) (2%) HEAPSORT

   (e) (2%) QUICKSORT

8. Consider the following weighted graph.



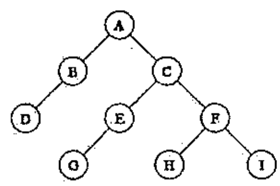
   (a) (5%) Show the order in which the edges are added to the minimum cost spanning tree using Kruskal's algorithm. (Use weight to represent edges in your answer.)

   (b) (5%) Show the order in which the edges are added to the minimum cost spanning tree using Prim's algorithm. (Use weight to represent edges in your answer.)

9. Determine the cost (5%) and structure (9%) of an optimal binary search tree for a set of $n = 4$ distinct keys k_1, k_2, k_3, and k_4 in sorted order (so that $k_1 < k_2 < k_3 < k_4$) with the following probabilities:

i	0	1	2	3	4
p_i		0.20	0.20	0.10	0.10
q_i	0.10	0.15	0.05	0.05	0.05

10. (6%) Show the results of traversing the following binary tree in preorder, inorder, and postorder.



國立中山大學九十四學年度碩士班招生考試試題

科目：工程數學【資工系碩士班乙組選考】

1. A real matrix, in which all elements are real numbers, is *square* if the number of rows equals the number of columns. A square matrix S is *symmetric* if $S_{ij} = S_{ji}$, *anti-symmetric* if $S_{ij} = -S_{ji}$. The *trace* of a square matrix S, denoted by $tr(S)$, is the sum of all diagonal elements. A square matrix is *stochastic* if all elements are non-negative and the elements in each row sum to 1. A stochastic matrix is *doubly stochastic* if its transpose is stochastic. Suppose that A is a real square matrix, and A^t is its transpose.

   (a) (5%) Show that if $tr(A^tA) = 0$, then $A = 0$.

   (b) (5%) Show that if $A^tA = A^2$, then A is symmetric.

   (c) (5%) Show that A can be expressed as the sum of a symmetric matrix and an anti-symmetric matrix.

   (d) (5%) Show that for an $n \times n$ doubly stochastic matrix P, the n-dimensional vector

   $$u = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$$

   satisfies $u = u\ P$.

2. (a) (10%) An impulse train,

   $$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

   where $\delta(t)$ is the Dirac delta function, is periodic so it can be represented as a Fourier series. Compute the coefficients of the Fourier series of $s(t)$ in exponential form and show that

   $$s(t) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} e^{jk\omega_s t},$$

   where $\omega_s = \frac{2\pi}{T_s}$.

   (b) (5%) The Fourier transform (FT), which is defined by

   $$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt,$$

   of 1 is $2\pi\delta(\omega)$. Use this fact to show that

   $$\sum_{n=-\infty}^{\infty} \delta(t - nT_s) \ \xrightarrow{FT} \ \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s).$$

   (c) (10%) Let $x_s(t) = x_c(t)s(t)$ be the periodically sampled version of a continuous-time function $x_c(t)$. Use the convolution theorem or otherwise to show that

   $$X_s(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X_c(\omega - k\omega_s),$$

   where $X_s(\omega), X_c(\omega)$ are respectively the Fourier transforms of $x_s(t), x_c(t)$.

3. The *discrete Fourier transform* (DFT) of an N-point finite duration sequence, $\{x[n]; 0 \le n \le N-1\}$, is defined by

   $$X[k] = \sum_{n=0}^{N-1} x[n] W^{kn},$$

   where $W = e^{-j(\frac{2\pi}{N})}$. The *inverse* discrete Fourier transform (IDFT) is defined by

   $$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W^{-kn}.$$

   (a) (10%) Show that the IDFT of the DFT of an N-point finite-duration sequence $\{x[n]\}$ is periodic with period N.

   (b) (10%) Direct computation of N-point DFT is of complexity $O(N^2)$. However, the computation of an N-point DFT, where $N = LM$, can be implemented by M computations of L-point DFT followed by L computations of M-point DFT. Show that the time complexity of computing N-point DFT is reduced to $O(N \log N)$ with this implementation. Note that you do *not* need to specify this implementation.

4. A student answers N questions in a test, with

$$Pr(N = n) = \frac{\lambda^n}{n!}e^{-\lambda},$$

where $\lambda > 0$. Suppose that each question is correctly answered with probability p, independently of the other questions. Let K be the number of correctly answered questions by this student. Note that both N and K are random variables and that $0 \leq K \leq N$. Show that

(a) (5%) $E(N) = \lambda$

(b) (5%) $E(K|N) = pN$

(c) (5%) $E(K) = p\lambda$

(d) (5%) $E(N|K) = K + (1-p)\lambda$

5. (15%) The cumulative distribution function $F(x) = Pr(X \leq x)$ of a continuous random variable X can be written as

$$F(x) = \int_{-\infty}^{t} f(t)dt,$$

where $f(x)$ is called the probability density function of X.

Given a random sample generator G for a random variable U, whose density function is 1 in the interval of $[0, 1]$ and 0 otherwise, one can produce random samples for another random variable X with distribution $f(x)$ by a transformation $X = F^{-1}(U)$. Suppose that X is exponentially distributed with parameter $\lambda > 0$,

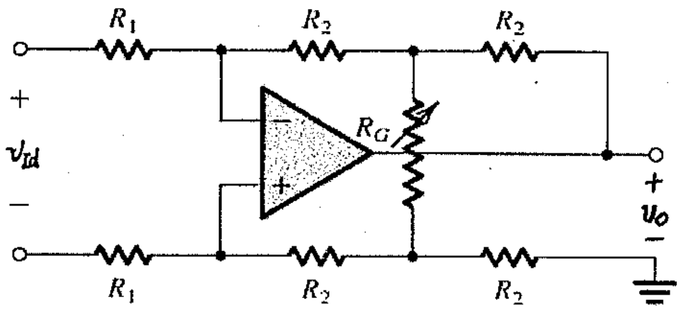
$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & x < 0. \end{cases}$$

Describe how one can create samples for X via G. In particular, all relevant functions must be specified in explicit functional forms.

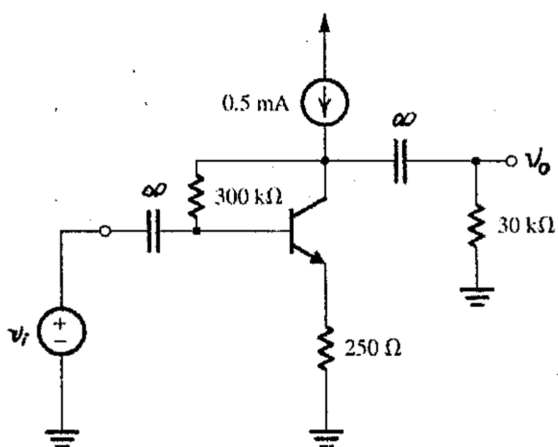
1. (20%) (a) (10%) Prove the unity gain frequency f_T of a MOSFET,

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})},$$ (b) (10%) Showing that $f_T \approx \frac{1.5}{\pi L}\sqrt{\frac{\mu_n I_D}{2C_{ox}WL}}$ by making the approximation that $C_{gs} >> C_{gd}$.

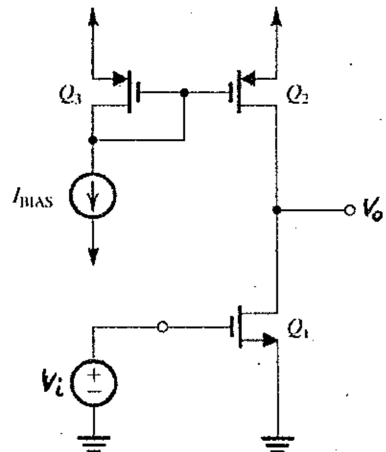
2. (20%) The following picture is a modified version of the difference amplifier. Show that the differential voltage gain is given by $\frac{v_O}{v_{Id}} = -2\frac{R_2}{R_1}[1+\frac{R_2}{R_G}]$



3. (20%) The BJT in the following circuit has β=100.
(a)(5%) Find the dc collector current. (b)(5%) Find the dc voltage at the collector. (c)(10%) Replacing the transistor by its T model, draw the small-signal equivalent circuit of the amplifier. Analyze the resulting circuit to determine the voltage gain v_o/v_i



4. (20%) A CMOS amplifier is shown in the following picture. The dc bias current is 100μA, For Q_1, $\mu_n C_{ox}$=90μA/V^2, V_A=12.8V, W/L=100μm/1.6μm, C_{gs}=0.2pF, C_{gd}=0.015pF, and C_{db}=20fF. For Q_2, C_{gd}=0.015pF, C_{db}=36fF, and $|V_A|$=19.2V. Assume that the resistance of the input signal generator is negligibly small. Also, for simplicity assume that the signal voltage at the gate of Q_2 is zero. Find the low-frequency gain(10%), the frequency of the pole(5%), and the frequency of the zero(5%).



5. (20%) The BJT in the following circuit has β=100, $|V_{BE}| = 0.7V$, $r_o = \infty$.

(a) (6%) For inputs grounded and output held at 0V(by negative feedback, not shown), find the emitter currents of all transistors. (b) (7%) Calculate the dc gain of the amplifier with $R_L = 10k\Omega$. (c) (7%) With R_L as in (b), find the value of C_C to obtain a 3-dB frequency of 100Hz?