國 立 清 華 大 學 命 題 紙

98 學年度 資訊系統與應用研發 (所) 丙 組碩士班入學考試

科目 離散數學 科目代碼 2201 共 2 頁第 1 頁 *請在【答案卷卡】內作答

1. (10%) If n is a positive integer and $n > 1$, prove that $C(n, 2) + C(n-1, 2)$ is a perfect square.

2. (10%) In how many ways can a dozen apples be distributed among five children so that no child gets more than seven apples?

3. (10%) Let $A, B, C \subseteq U$. Prove that $(A \cap B) \cup C = A \cap (B \cup C)$ if and only if $C \subseteq A$.

4. (10%) Please answer "True" or "False" for the following statements about finite state machines.

(4a) The language $L = \{a^k b^k \mid k \geq 1\}$ is a finite state language. (2 points)

(4b) The language $L = \{0^k \mid k = 2^i, i \geq 1\}$ is not a finite state language. (2 points)

(4c) The language $L = \{(01)^i 1^{2j} \mid i \geq 1, j \geq 1\}$ is not a finite state language. (2 points)

(4d) Nondeterministic finite state machines are more powerful than deterministic finite state machines in the sense that there are languages that can be recognized by a nondeterministic machine but cannot be recognized by a deterministic one. (2 points)

(4e) In the following finite state machine, C and F are equivalent states, D and G are equivalent states, and E and H are equivalent states. (2 points)

FA	Input		
State	0	1	Output
$\rightarrow A$	B	C	0
B	F	D	0
C	G	E	0
D	H	B	0
E	B	F	1
F	D	H	0
G	E	B	0
H	B	C	1

5. (10%) Please answer "True" or "False" for the following statements about trees.

(5a) Any edge of a connected graph G is a chord of some spanning tree of G. (2 points) (Note: A *chord* of a tree is an edge of the graph that is not in the tree. The set of the chords of a tree is referred to as the complement of the tree.)

(5b) Any edge of a connected graph G is a branch of some spanning tree of G. (2 points)

(5c) There are 15 spanning trees in a complete graph with 4 distinctly labeled vertices. (2 points)

(5d) There must be a cut-set that has no common edge with a spanning tree. (2 points)

(5d) For a regular n-ary tree, we have $E = (n - 1) I + ni$, where i denotes the number of branch nodes, E denotes the sum of the path lengths of the leaves in a rooted tree, and I denotes the sum of the path lengths of all the branch nodes. (2 points)

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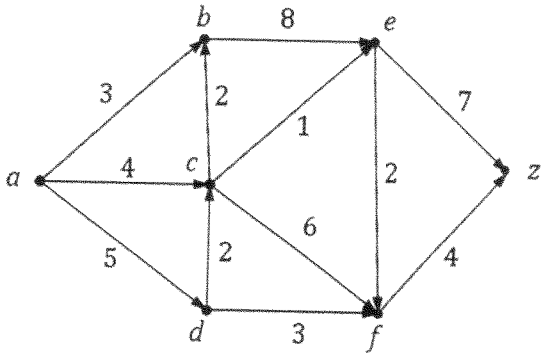
6. (8%)

(6a) Find a minimum cut of the following transport network. Use the notation (S, T) to denote a cut that divides the vertices into two subsets S and T, where the subset S contains the source and the subset T contains the sink. (5 points)

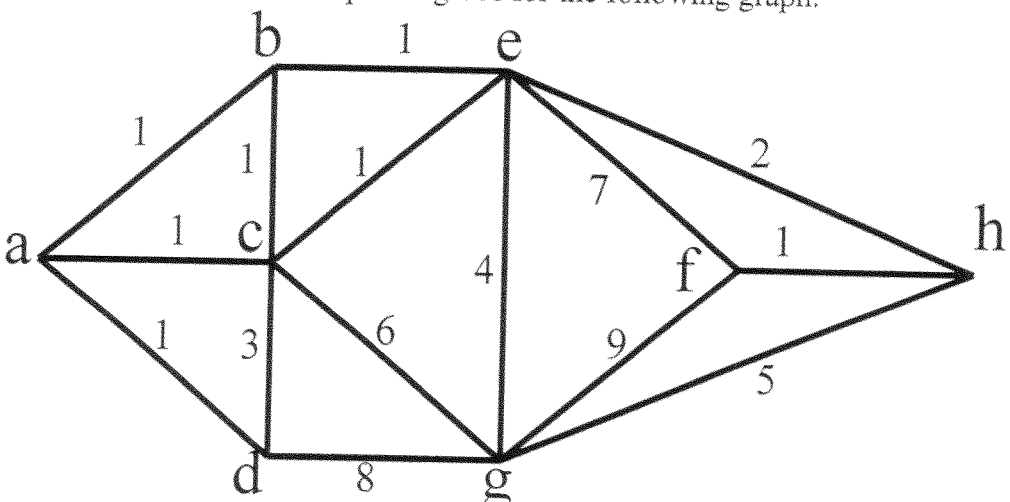
(6b) What is the largest possible value of a flow in this transport network? (3 points)



7. (7%) Assume that n cities are connected by a network of k highways. (A highway is defined to be a road between two cities that does not go through any intermediate cities.) Show that if $k > \frac{1}{2}(n-1)(n-2)$, then one can always travel between any two cities through connecting highways.

8. (7%) Use the procedure invented by Huffman to construct an optimal tree for a given set of weights: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

9. (8%) Determine a minimum spanning tree for the following graph.



10. (10%) What is the generating function for $a_r = 3^r$, $r \geq 0$? Use the method of generating functions to find the convolution $\boldsymbol{a}^*\boldsymbol{b}$, where $a_r = 3^r$ and $b_r = 2^r$, $r \geq 0$.

11. (10%) Derive an explicit expression for Fibonacci numbers F_i, which satisfy $F_0 = 0, F_1 = 1$, and $F_i = F_{i-1} + F_{i-2}$, *for* $i \geq 2$.