1. (a) What is the Pigeonhole Principle? (5%)
   (b) Show that if any five numbers from 1 to 8 are chosen, then two of them will add up to 9. (5%)

2. Give a grammar that specifies the language $\{(ab)^k c^{2j} \mid k,j \geq 1\}$. (10%)

3. Answer the following questions related go graph. We assume that a graph contains no multiple edges between two vertices, and contains no self-loop. Briefly explain how your answer is derived.
   (a) What is the minimum number of vertices a graph can have if the graph has 100 edges? (5%)
   (b) What is the maximum number of edges a graph can have if the graph has 100 vertices and the graph is bipartite? (5%)

4. Assume the functions f, g, and h take on only positive values. Determine whether the following statement is true or false. If the statement is false, give a counterexample:
   (a) If $f(n)=\Theta(g(n))$ and $g(n)=\Theta(h(n))$, then $f(n)=\Theta(h(n))$ (4%)
   (b) If $f(n)=\Theta(h(n))$ and $g(n)=\Theta(h(n))$, then $f(n)+g(n)=\Theta(h(n))$ (4%)
   (c) If $f(n)=\Theta(g(n))$ then $cf(n)=\Theta(g(n))$ for any $c\neq 0$ (4%)
   (d) If $f(n)=\Theta(g(n))$, then $2^{f(n)}=\Theta(2^{g(n)})$ (4%)
   (e) If $f(n)=O(g(n))$, then $g(n)=O(f(n))$ (4%)

5. Given a encoding function $e: B^2 \to B^5$ as follows: (20%)

$$e(00)=00000$$
$$e(01)=01111$$
$$e(10)=10101$$
$$e(11)=11010$$

and the parity check matrix

$$H=\begin{bmatrix}1&0&1\\1&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

we receive the word $x=01101$
   (a) Find the coset leader of x, and decode x.
   (b) Suppose we know that 00000, 00001, and 00010 are coset leaders. Compute the syndrome of each coset leader, and decode x.

6. Try to use (a) **characteristic polynomial method** and (b) **generating function method** to solve the recurrence relation: (20%)

$$a_n = 2a_{n-1}+1, a_0 = 1$$

7. $f: G \to G$ is homomorphic,, $H=\{a \in G \mid f(a)=a\}$, prove that H is a subgroup of G. (10%)

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